3. LUR'YE A.I., Non-linear Elasticity Theory. Nauka, Moscow, 1980.
4. MUSKHELISHVILI N.I., Certain Fundamental Problems of the Mathematical Theory of Elasticity. Nauka, Moscow, 1966.
5. MUSKHELISHVILI N.I., Singular Integral Equations. Nauka, Moscow, 1968.

Translated by M.D.F.

PMM U.S.S.R., Vol.53, No.5, pp.643-649, 1989
Printed in Great Britain

0021-8928/89 $10.00+0.00

THE CONSTRUCTION OF THE DISSIPATIVE PLASTIC FLOW FUNCTION ON THE BASIS OF MICROSCOPIC REPRESENTATIONS*

V.N. LUKERCHENKO

A dissipative function (DF) of the plastic flow of a single crystal is constructed on the basis of microscopic representations. A thermodynamic analysis is performed of the possible mechanical energy dissipation mechanism for moving dislocations. The general expression constructed for the DF is reduced to a form such that the latter depends only on characteristics of the process (strain rates) and macroscopic characteristics of the ensemble of dislocations. The physical meaning is uncovered here and the value of all the coefficients in the determination of the DF is indicated. The deduction is made that the phenomenological representation of the DF just as the sum of first and second degree homogeneous functions in the plastic strain rates is generally non-uniform and the rate of change of the mocrostructure parameters must still be taken into account.

The construction of the dissipative function (DF)

$$\Phi = T^{-1}dq'/dt \tag{0.1}$$

(where T is the absolute temperature, q' is the uncompensated heat, and t is the time) governing the magnitude of entropy growth due to internal irreversible processes is the most important element in describing plastic deformation and the construction of new models of continuous media /1, 2/. Usually it is postulated phenomenologically that the DF for plastic media is a homogeneous (linear or non-linear) function of first degree in the plastic strain rates ε_{ij} while it is a homogeneous second-degree function in the plastic strain rates or the sum of the above-mentioned first and second degree homogeneous functions for viscoplastic media /3, 4/. It is impossible to regard such an approach as completely satisfactory for the following reasons. 1°. It is assumed that the coefficients in the determination of the homogeneous functions can be determined experimentally. As a rule, however, the appropriate experimental data have a large spread. 2°. The coefficients mentioned are not determined from physical representation, i.e., on the basis of the material microstructural characteristics, whereupon their physical meaning is also not clear. 3°. It is also not known whether a DF of a plastic medium with dislocations can be constructed just like a homogeneous function (or the sum of homogeneous functions) in the plastic strain rates without taking account of the microstructural parameters of the plastic flow and their rate of change.

The DF was introduced in the form of the general expression (/5-7/, et al.)

$$\Phi = \Phi(\varepsilon_{ij}', T, \mu, \mu')$$

when taking account of the internal parameters and their rates of change, where μ is an internal parameter. However, the form of the DF is not made specific here and the proposed models were not properly microscopic.

*Prikl.Matem.Mekhan., 53, 5, 816-823, 1989

1. Fundamental assumptions and initial microstructure. Let us consider an infinite homogeneous BCC or HCP-type single crystal oriented for single slip. We shall consider strain to occur at medium temperatures and loading rates while the transverse slip of the screw elements of the dislocations do not exist (as is justified at the initial stage of strain and especially for single crystals with a low stacking energy). We assume the coordinate system to be rectangular, and the space Euclidean.

The manifold of mobile curvilinear dislocation structures in the crystal reduces to two main types: loops and segments that are characterised, in turn, by the Burgers vector $\mathbf{b}$ and a mean non-local radius of curvature R /8/. Some of the dislocations are fixed: fixed basic growing dislocations with density β and Burgers vector $\mathbf{b}$ and fixed growing scaffold dislocations with density β_* and Burgers vector $\mathbf{b}_{(*)}$ The slip plane of the mobile dislocations is characterised by the unit normal vector $\mathbf{n}$ while the orientation of the dislocation element is characterised by the unit tangent vector $\boldsymbol{\xi}$ (or the angle φ between $\mathbf{b}$ and $\boldsymbol{\xi}$).

2. Fundamental equations of the model. Randomly distributed in parallel slip planes per unit volume of the crystal are N_0 dislocation segments whose distribution density over the lengths of the base λ corresponds to the distribution density of the spacings between points scattered randomly on the line

$$\varphi(\lambda) = \lambda\lambda_0^{-2}\exp(-\lambda\lambda_0^{-1}) \tag{2.1}$$

where $2\lambda_0 = \langle\lambda\rangle_\lambda$. Here and below the symbols $\langle\ldots\rangle_D$, $\langle\ldots\rangle_\lambda$, and $\langle\ldots\rangle_R$, respectively, denote the mean along the dislocation line, the mean in the ensemble of segments, and the mean in the ensemble of loops. Under certain conditions, the mobile segments whose base length λ can vary from the minimum value λ_t to infinity emit dislocation loops whose initial radius is R_λ The segment length $l(\lambda, t)$ with base length λ that changes in the interval $[\lambda, 2\pi R_\lambda + \lambda]$ can be characterized by the function $\omega(\lambda, t) = l(\lambda, t)(2\pi R_\lambda + \lambda)^{-1}$. The area marked by the segment with the base length λ per unit time will be considered to be equal to $\pi R_\lambda^2\eta^\cdot(\lambda, t)$ where $\eta(\lambda, t)$ has the meaning of the area marked by a segment at the time t to the initial area πR_λ^2 of the loop emitted by this segment. The characteristics $\omega(\lambda, t)$ and $\eta(\lambda, t)$ are connected by the relationship

$$\pi R_\lambda^2\eta^\cdot(\lambda, \chi) = (2\pi R_\lambda + \lambda)\,\omega(\lambda, \chi)\langle v(\lambda, \chi)\rangle_D \tag{2.2}$$

where $\langle v(\lambda, \chi)\rangle_D$ is the mean of the segment velocity along the length at the time χ.

The total length per unit volume of the segments $A_*(t)$ is comprised of lengths of fixed segments with base length $\lambda \leqslant \lambda_t$ and moving segment lengths with total density $\alpha_*(t)$.

$$A_*(t) = N_0\int_0^{\lambda_t}\lambda\varphi(\lambda)\,d\lambda + \alpha_*(t), \quad \alpha_*(t) = \int_{\lambda_t}^{\infty} l(\lambda, t)N_0\varphi(\lambda)\,d\lambda \tag{2.3}$$

Taking into account that $(2\pi R_{\lambda_t} + \lambda_t)\,\omega(\lambda_t, t) = l(\lambda_t, t) = \lambda_t$, $l^\cdot(\lambda, t) = 2\pi R_\lambda\eta^\cdot(\lambda, t)$ we obtain for the elementary increment of the segment density

$$dA_* = \int_{\lambda_t}^{\infty} 2\pi R_\lambda\eta^\cdot(\lambda, t)N_0\varphi(\lambda)\,d\lambda\,dt \tag{2.4}$$

Using the operator $\langle X\rangle_\lambda$ the mean length of the dislocation curve of the moving segments can be determined

$$\langle l(\lambda, t)\rangle_\lambda = \int l(\lambda, t)\varphi(\lambda)\,d\lambda\left(\int\varphi(\lambda)\,d\lambda\right)^{-1}, \quad \langle X\rangle_\lambda = \int X\varphi(\lambda)\,d\lambda\left(\int\varphi(\lambda)\,d\lambda\right)^{-1} \tag{2.5}$$

Here and henceforth, it is assumed that the integration over λ is between λ_t and ∞. The crystal loading law must be given for the further evaluation of $\langle l(\lambda, t)\rangle_\lambda$.

In addition to the segments, $N(t)$ of the mobile dislocation loops radius distribution function density $\Psi(R, t)$ and radii from the minimum r_t to the maximum R_t is found at the time t per unit crystal volume in parallel planes of single slip. The function $\Psi(R, t)$ is non-zero just for $r_t < R < R_t$ and is normalized to one

$$\int\Psi(R, t)\,dR = 1 \tag{2.6}$$

Here and henceforth, the radius R is understood to be the mean non-local radius of curvature and it is also assumed that integration over R is everywhere between r_t and R_t.

An increase in the density of the slipping dislocation loops occurs because of broadening

of the loops present earlier in the crystal, and due to newly generated loops. Loops with the identical mean non-local radius of curvature R can have a distinct mode configuration and size but on the average we shall approximately assume that the length of the mentioned loops equals the length of circular loops of the same radius. Then the sliding loop density will equal

$$\alpha_0(t) = 2\pi \langle R \rangle N(t) = 2\pi N(t) \int R\Psi(R, t)\, dR \tag{2.7}$$

Taking into account that $\Psi(R_l, t) = \Psi(r_l, t) = 0$ and $\langle v \rangle_D = R^{\cdot}$, we obtain the following expression for the growth rate of the mobile loop density:

$$\alpha_0^{\cdot}(t) = 2\pi N^{\cdot}(t) \int R\Psi(R, t)\, dR + 2\pi N(t) \int (\langle v \rangle_D \Psi(R, t) + R\Psi^{\cdot})\, dR \tag{2.8}$$

We furthermore assume that the dislocation motion equation

$$\varepsilon_{jkl}\xi_k n_l \sigma_{dm}^{(a)} b_{dm} = \varepsilon_{jkl}\xi_k n_l \langle \sigma_{dm}^{(*)} \rangle b_{dm} + \langle F_j^{(s)} \rangle_D + F_j^{(P)} + B \langle v_j \rangle_D \tag{2.9}$$

$$\langle \sigma_{dm}^{(*)} \rangle = \sqrt{2}\, b^{-1} b_{(dm)} (A_1 Gb \sqrt{\beta + \alpha_0(t)} + A_* (t) + A_2 Gb_{(*)} \sqrt{\beta_*})$$

$$b = |\mathbf{b}|, \; b_{dm} = b_d n_m, \quad b_{(dm)} = {}^1/_2 (b_d n_m + b_m n_d)$$

is valid for the component $\langle v_j(R, t) \rangle_D$ of the mean velocity of motion of the dislocation element with tangent vector ξ.

Here $\sigma_{dm}^{(a)}$ and $\langle \sigma_{dm}^{(*)} \rangle$ are components of the external applied stress and the mean stress (over the crystal) needed to push the dislocation element through the ensemble of adjacent dislocations, $F_j^{(p)}$ are the components of the Peierls dynamic force $(F^{(P)} = |\mathbf{F}^{(P)}|)$, B is the damping factor, G is the shear modulus, A_1 and A_2 are constants, and ε_{jkl} are the Levi-Civita symbols $(\varepsilon_{jkl} v_j \xi_k n_l \geqslant 0$ and $j, k, l, d, m = 1, 2, 3)$.

The force of the mean non-local selfaction of the dislocation structures whose components are /8/

$$\langle F_j^{(\cdot)} \rangle_D = \varepsilon_{jkl}\xi_k n_l AGb^2/R$$

also occurs in the relationship (2.9), where $A \approx {}^1/_2$. For a dislocation segment with base length λ the absolute value of the force $\langle F^{(\cdot)} \rangle_D = \langle |\mathbf{F}^{(\cdot)}| \rangle_D$ varies between $AGb^2/({}^1/_2\lambda)$ and AGb^2/R_λ during the whole evolution of the segment up to emission of the loop, we hence assume that

$$\langle \langle F^{(s)}(\lambda, \chi) \rangle_D \rangle_\chi = AGb^2/({}^5/_6\lambda) \tag{2.10}$$

where χ is the time, which varies in the interval between two dislocation loop emission times.

3. Relation of the dislocation ensemble characteristics to the magnitude of the crystal plastic strain rate. Because the crystal plastic strain $\varepsilon_{ij}(t)$ is proportional to the total area marked by the moving loops and segments /8/, we have

$$\varepsilon_{ij}(t) = b_{(ij)} \int \pi R^2 N(t)\, \Psi(R, t)\, dR + b_{(ij)} \int \pi R_\lambda^2 \eta(\lambda, t)\, N_0 \varphi(\lambda)\, d\lambda \tag{3.1}$$

where the first term on the right-hand side of (3.1) yields the contribution from the loop motion and the second from the segment motion. We furthermore assume that the area marked by the segment at the starting time equals zero, i.e., $\eta(\lambda_l, t) = 0$. Taking into account also that $\Psi(R_l, t) = \Psi(r_l, t) = 0$ we differentiate (3.1) and we obtain

$$\varepsilon_{ij}^{\cdot}(t) = b_{(ij)}\, 2\pi \int N(t)\, R \langle v(R, t) \rangle_D\, \Psi(R, t)\, dR + \tag{3.2}$$

$$b_{(ij)} \int \pi R^2 (N(t)\, \Psi(R, t))^{\cdot}\, dR + b_{(ij)} \int \pi R_\lambda^2 \eta^{\cdot}(\lambda, t)\, N_0 \varphi(\lambda)\, d\lambda$$

where the first two terms on the right-hand side of (3.2) yield the contribution to the quantity $\varepsilon_{ij}^{\cdot}$ from the loop motion, and the third term is the contribution from the segment motion.

4. Fundamental mechanisms of the dissipation of moving dislocation mechanical energy. We examine below the fundamental mechanical energy dissipation mechanisms of moving

dislocations into crystal thermal energy. We first write the expression for the total magnitude of the increments of the moving dislocation structure energy being dissipated (the loops and segments) per unit crystal volume

$$dq' = \int_0^{2\pi}\int (B\langle v\rangle_D)\langle v\rangle_D RN(t)\Psi(R,t)\,dR\,d\varphi\,dt + \int_0^{2\pi}\int F^{(P)}\langle v\rangle_D RN(t)\rangle \tag{4.1}$$

$$\Psi(R,t)\,dR\,d\varphi\,dt + \int_0^{2\pi}\int |\langle\sigma_{dm}^{(*)}(t)\rangle b_{dm}|\langle v\rangle_D RN(t)\Psi(R,t)\,dR d\varphi\,dt +$$

$$\int (|\sigma_{dm}^{(a)} b_{dm}| - \langle\langle F^{(s)}\rangle_D\rangle_\chi)\pi R^2 d(\Psi(R,t)N(t))\,dR + \int (F_*^{(P)} + B\langle v\rangle_D)\rangle$$

$$\pi R_\lambda{}^2\eta^*(\lambda,t)N_0\varphi(\lambda)\,d\lambda\,dt, \quad F_*^{(P)} = F^{(P)} + |\langle\sigma_{dm}^{(*)}(t)\rangle b_{dm}|$$

We will clarify the physical meaning of the quantities on the right-hand side of (4.1). The first integral on the right-hand side of (4.1) characterizes the dissipation mechanical energy of dislocation loops into thermal energy of the crystal because of viscous deceleration. The dissipation occurs by the appearance of phonon viscosity, thermoelastic dissipation, phonon scattering, and flutter-effect mechanisms, where all the mechanisms mentioned result in viscous deceleration of the dislocations, which depend in a linear manner on the dislocation rate.

The second integral on the right-hand side of (4.1) describes the loop mechanical energy dissipation due to the action of radiation friction (the dynamic Peierls force) that a dislocation experiences during its displacement over a discrete crystalline lattice when its atomic configuration and elastic energy experience periodic changes. The dislocation here emits phonons (thermal oscillations), i.e., gives up part of its energy to the atoms of the crystalline lattice.

The third integral on the right-hand side of (4.1) corresponds to loop mechanical energy dissipation due to their deceleration by the internal long-acting stress fields of adjacent dislocations. This dissipation mechanism is realized because of arching of the moving dislocation arcs between peaks of the field $\sigma_{dm}^{(*)}$, their collapse and partial annihilation with the adjacent arcs during rectification of the dislocations. Then this process is repeated again and again. It should be noted that a thermal energy increment is possible here even because of the formation of cutoff processes during intersection of the dislocations and deceleration of the dislocations because of the non-conservative motion of these cutoffs. However, these effects are only important at low temperatures ($T < 0.2T_0$, where T_0 is the absolute melting point) and low velocities of dislocation motion.

The fourth integral on the right-hand side of (4.1) has the meaning of an increment of the crystal thermal energy due to dissipation of the mechanical energy of the moving dislocation loops, occuring in the crystal during the time interval between t and $t + dt$, into thermal energy over all possible channels.

Let us examine the last integral on the right-hand side of relationship (4.1). Each of the $N_0\varphi(\lambda)\,d\lambda$ dislocation segments with base length from λ to $\lambda + d\lambda$ in the time interval dt marks an area $\pi R_\lambda{}^2\eta^*(\lambda, t)\,dt$ that performs work against the radiation friction force, the internal remote-acting stress fields of the adjacent dislocations and the viscous deceleration. Thus, the last integral on the right-hand side of (4.1) characterizes the mechanical energy dissipation of moving dislocation segments over all the channels listed above.

The microscopic characteristics of individual dislocation structures occur in (4.1). We later transform this relationship into a form when only macroscopic plastic flow characteristics and statistical characteristics of the ensemble of dislocation loops and segments will occur on its right-hand side.

5. An expression for the dissipative function. We first transform the second and third integrals on the right-hand side of (4.1), as well as those parts of the fourth and fifth integrals that characterize energy dissipation due to radiation friction and internal long-acting stress fields of adjacent dislocations. We here use relationships (3.2), having first convoluted it with $2b_{(ij)}b^{-2}$, and also the equality

$$2b_{(ij)}b_{(ij)} = b^2 \tag{5.1}$$

We hence have

$$\int_0^{2\pi}\int F_*^{(P)}\langle v\rangle_D RN(t)\Psi(R,t)\,dR\,d\varphi\,dt + \int_0^{2\pi}\int F_*^{(P)}\pi R^2 d(\Psi(R,t)N(t))\,dR + \tag{5.2}$$

$$\int F_*^{(P)}\pi R_\lambda{}^2\eta^*(\lambda,t)N_0\varphi(\lambda)\,d\lambda\,dt = F_*^{(P)}2b_{(ij)}\varepsilon_{ij}^*(t)\,b^{-2}\,dt$$

We now transform the first integral on the right-hand side of (4.1) and that part of the fourth integral that characterizes mechanical energy dissipation due to viscous deceleration of the dislocations. We have

$$J_* = \int_0^{2\pi} \int B \langle v \rangle_D \langle v \rangle_D RN(t) \Psi(R, t)\, dR\, d\varphi\, dt + \int B \langle \langle v \rangle_D \rangle_\chi \pi R^2 d(\Psi(R, t) N(t))\, dR$$

Furthermore, using the value $\langle v \rangle_D$ from (2.9), first convoluted with the unit vector with the components $\xi_{jkl}\xi_k n_i$ and taking (2.10) into account, we find the mean value of the dislocation segment velocity in the time interval $t_n(\lambda) \leqslant \chi \leqslant t_{n+1}(\lambda)$ i.e., the value $\langle \langle v \rangle_D \rangle_\chi$ in the expression for J_*

$$\langle \langle v(\lambda, t) \rangle_D \rangle_\chi = (t_{n+1}(\lambda) - t_n(\lambda))^{-1} \int_{t_n^{(\lambda)}}^{t_{n+1}^{(\lambda)}} \langle v(\lambda, t) \rangle_D\, d\chi =$$

$$(t_{n+1}(\lambda) - t_n(\lambda))^{-1} \int_{t_n^{(\lambda)}}^{t_{n+1}^{(\lambda)}} B^{-1}(K - \langle F^{(\cdot)}(\lambda, t) \rangle_D)\, d\chi = B^{-1}(K - AGb^2/(^5/_6\lambda)),$$

$$K = |\sigma_{dm}^{(a)}(t)\, b_{dm}| - \langle \sigma_{dm}^{(*)}(t) \rangle\, b_{dm} - F^{(P)}$$

Moreover, using relationship (2.6) and the properties of the distribution density function $\Psi(R, t)$ and also taking into account that the mobile dislocation loop density $\alpha_0(t)$ is given by relationship (2.7), while the values of the mean velocity of the ensemble of dislocation loops $\langle v \rangle_R$ and the velocity of the dislocation loop of mean radius $v_{\langle R \rangle_R}$ is found from the equalities

$$B \langle v \rangle_R = K - AGb^2 \langle R^{-1} \rangle_R, \quad B v_{\langle R \rangle} = K - AGb^2 \langle R \rangle_R^{-1} \tag{5.3}$$

(here $v_{\langle R \rangle_R} \neq \langle R \rangle_R^{\cdot}$), we obtain

$$J_* = \alpha_0(t)\, dt\, (K v_{\langle R \rangle_R} - AGb^2 \langle v \rangle_R \langle R \rangle_R^{-1}) + K \int \pi R^2 d(\Psi(R, t) N(t))\, dR - AGb^2 \int 2\pi R\, d(\Psi(R, t) N(t))\, dR \tag{5.4}$$

The expression $v_{\langle R \rangle_R}$ in terms of the plastic strain rate tensor components $\varepsilon_{kl}^{\cdot(0)}$ that occur only because of the motion and multiplication of the dislocation loops must be found for the subsequent calculations. It follows from (3.2) that

$$\varepsilon_{il}^{\cdot(0)}(t) = I_1 + I_2 \tag{5.5}$$

$$I_1 = b_{(il)} 2\pi \int \langle v \rangle_D RN(t) \Psi(R, t)\, dR, \quad I_2 = b_{(kl)} \int \pi R^2 (\Psi(R, t) N(t))^{\cdot} dR$$

We transform the first integral on the right-hand side of this equality. Using relationships (2.7), (2.9), and (5.3), we have

$$I_1 = 2\pi N(t) B^{-1} b_{(kl)} \int (K - AGb^2 R^{-1}) R \Psi(R, t)\, dR = 2\pi N(t) \langle R \rangle_R v_{\langle R \rangle_R} b_{(kl)} = b_{(kl)} \alpha_0(t) v_{\langle R \rangle_R}$$

Substituting this value into (5.5) and convoluting it with $2b^{-2}b_{(kl)}$ taking (5.1) into account, a value can be found for the velocity of a loop having a mean radius in the ensemble of loops

$$v_{\langle R \rangle_R} = \frac{2b_{(kl)}\varepsilon_{kl}^{\cdot(0)}(t)}{b^2 \alpha_0(t)} - \frac{1}{\alpha_0(t)} \int \pi R^2 (\Psi(R, t) N(t))^{\cdot} dR \tag{5.6}$$

Substituting the value $v_{\langle R \rangle_R}$ from (5.6) into (5.4), and also taking account of (2.7), (2.9), and (5.3), we obtain

$$J_* = \left(\frac{4Bb_{(ij)} b_{(kl)} \varepsilon_{kl}^{\cdot(0)}(t)}{b^2 \alpha_0(t)} - \frac{2Bb_{(ij)}}{b^2 \alpha_0(t)} \int \pi R^2 (\Psi(R, t) N(t))^{\cdot} dR + \frac{2AGb^2 b_{(ij)}}{b^2 \langle R \rangle_R} \right) d\varepsilon_{ij}^{(0)} - AGb^2\, d\alpha_0 \tag{5.7}$$

We now transform that part of the last integral in (4.1) that characterizes the energy dissipation due to viscous deceleration of the moving dislocation segments. Using (2.2) and (2.9) written for dislocation segments in scalar form

$$B\langle v(\lambda,t)\rangle_D = K - \frac{AGb^2\dot{l}(\lambda,t)}{\langle v(\lambda,t)\rangle_D\, l(\lambda,t)} \tag{5.8}$$

we obtain

$$J_{**} = \int B\langle v\rangle_D \pi R_\lambda^2 \dot{\eta}(\lambda,t) N_0\varphi(\lambda)\, d\lambda\, dt = \tag{5.9}$$
$$\int\left(K - \frac{AGb^2 l(\lambda,t)}{\langle v(\lambda,t)\rangle_D\, l(\lambda,t)}\right) l(\lambda,t)\langle v(\lambda,t)\rangle_D N_0\varphi(\lambda)\, d\lambda\, dt$$

Furthermore, remarking that it follows from (3.2) that the plastic strain rate tensor components of a crystal that occur just because of dislocation segment motion are

$$\varepsilon_{ij}^{(*)}(t) = b_{(ij)}\int \pi R_\lambda^2 \dot{\eta}(\lambda,t) N_0\varphi(\lambda)\, d\lambda = b_{(ij)}\int l(\lambda,t)\langle v(\lambda,t)\rangle_D N_0\varphi(\lambda)\, d\lambda \tag{5.10}$$

and using the relationships (2.4) and (5.1), we obtain

$$J_{**} = 2b_{(ij)}b^{-2}K\dot{\varepsilon}_{ij}^{(*)}(t)\, dt - AGb^2\, dA_* \tag{5.11}$$

We now note that it follows from (5.10) that

$$\dot{\varepsilon}_{kl}^{(*)}(t) = b_{(kl)}\langle l\langle v\rangle_D\rangle_\lambda M(t) = b_{(kl)}\alpha_*(t)\langle l\langle v\rangle_D\rangle_\lambda\langle l\rangle_\lambda^{-1} \tag{5.12}$$

Furthermore, applying the operator $\langle\ldots\rangle_\lambda$ to (5.8) and substituting the value obtained $\langle\langle v\rangle_D\rangle_\lambda$ from (5.8) into (5.12) and also using (5.1), we obtain

$$K = \frac{2Bb_{(kl)}\dot{\varepsilon}_{kl}^{(*)}(t)}{b^2\alpha_*(t)} - \frac{B\langle l(\langle v\rangle_D - \langle\langle v\rangle_D\rangle_\lambda)\rangle_\lambda}{\langle l\rangle_\lambda} + \langle\langle F^{(s)}\rangle_D\rangle_\lambda \tag{5.13}$$

Finally, substituting the value of K from (5.13) into (5.11), we find

$$J_{**} = \left(\frac{4Bb_{(ij)}b_{(kl)}\dot{\varepsilon}_{kl}^{(*)}(t)}{b^4\alpha_*(t)} - \frac{2Bb_{(ij)}\langle l(\langle v\rangle_D - \langle\langle v\rangle_D\rangle_\lambda)\rangle_\lambda}{b^2\langle l\rangle_\lambda} + \right. \tag{5.14}$$
$$\left. 2b_{(ij)}b^{-2}\langle\langle F^{(s)}\rangle_D\rangle_\lambda\right) d\varepsilon_{ij}^{(*)} - AGb^2\, dA_*$$

Now using the transformed expression of (4.1) for the increment of the total magnitude of the dissipated energy of the moving dislocation structures per unit volume of the crystal, referred to unit absolute temperature, an expression can be written for the DF of the plastic flow of a crystal

$$\Phi = L_{ij}\dot{\varepsilon}_{ij}^{(0)} + F_{ijkl}\dot{\varepsilon}_{kl}^{(0)}\dot{\varepsilon}_{ij}^{(0)} + M_{ij}\dot{\varepsilon}_{ij}^{(*)} + E_{ijkl}\dot{\varepsilon}_{kl}^{(*)}\dot{\varepsilon}_{ij}^{(*)} - \tag{5.15}$$
$$AGb^2T^{-1}\dot{\alpha}_0 - AGb^2T^{-1}\dot{A}_*$$

$$L_{ij} = N_{ij} - \frac{2Bb_{(ij)}}{Tb^2\alpha_0(t)}\int \pi R^2(\Psi(R,t)N(t))^{\cdot}\, dR + \frac{2AGb^2b_{(ij)}}{Tb^2\langle R\rangle_R}$$

$$F_{ijkl} = \frac{4Bb_{(ij)}b_{(kl)}}{Tb^4\alpha_0(t)}, \quad E_{ijkl} = \frac{4Bb_{(ij)}b_{(kl)}}{Tb^4\alpha_*(t)}$$

$$M_{ij} = N_{ij} - \frac{2Bb_{(ij)}\langle l(\lambda,t)(\langle v(\lambda,t)\rangle_D - \langle\langle v(\lambda,t)\rangle_D\rangle_\lambda)\rangle_\lambda}{Tb^2\langle l(\lambda,t)\rangle_\lambda} + \frac{2b_{(ij)}}{Tb^2}\langle\langle F^{(s)}(\lambda,t)\rangle_D\rangle_\lambda$$

$$N_{ij} = \frac{2F^{(P)}b_{(ij)}}{Tb^2} + \frac{\sqrt{2}\,b_{(ij)}}{Tb}\left(A_1Gb\sqrt{\beta + \alpha_0(t) + A_*(t)} + A_2Gb_{(*)}\sqrt{\beta_*}\right)$$

where $d\varepsilon_{ij}^{(0)}$ and $d\varepsilon_{ij}^{(*)}$ are, respectively, the increment of plastic strain due to loop motion and segment motion (the total plastic strain is $\varepsilon_{ij} = \varepsilon_{ij}^{(0)} + \varepsilon_{ij}^{(*)}$) It can be seen that the right-hand side of (5.15) is always non-negative. The relationship obtained shows that homogeneous functions of the first and second degree in the plastic strain rates $\dot{\varepsilon}_{ij}^{(0)}$ and $\dot{\varepsilon}_{ij}^{(*)}$ produced by both kinds of dislocation structures, occur in the expression for the DF. However, the essential distinction from phenomenological methods of giving the DF is the dependence on not only the plastic strain rates but also on the rates of change of the crystal

microstructure parameters α_0^* and A_*^*. The coefficients $L_{ij}, F_{ijkl}, M_{ij}, E_{ijkl}$ and AGb^2T^{-1} in (5.15) for the DF are expressed in terms of the macroscopic characteristics of the ensemble of dislocation structures and have a specific value and an explicit physical meaning, and can be determined from the solution of the equations of the model /8, 9/.

REFERENCES

1. SEDOV L.I., Mechanics of a Continuous Medium, 1, Nauka, Moscow, 1973.
2. SEDOV L.I., Mathematical methods of constructing new models of continuous media, Usp. Mat. Nauk, 20, 5, 1965.
3. IVLEV D.D., On the dissipative function in the theory of hardening plastic media, PMM, 31, 2, 1967.
4. IVLEV D.D., On the dissipative function in the theory of plastic media, Dokl. Akad. Nauk, SSSR, 176, 5, 1967.
5. KOLAROV D., BALTOV A. and BONCHEVA N., Mechanics of Plastic Media. Mir, Moscow, 1979.
6. PERZHINA P. and SAWCHUK A., Problems of thermplaticity. Problems of the Theory of Plasticity and Creep, Mir, Moscow, 1979.
7. MROZ Z. and RANIECKI B., A derivation of the uniqueness conditions in coupled thermo plasticity, Intern. J. Eng. Sci., 14, 4, 1976.
8. LUKERCHENKO V.N., On the derivation of the fundamental kinematic relationship of dislocation theory, Dokl. Akad. Nauk SSSR, 274, 1, 1984.
9. LUKERCHENKO V.N., On the theory of deformation hardening of crystalline materials, Fizika i Khimiya Obrab. Materialov, 4, 1983.

Translated by M.D.F.

PMM U.S.S.R., Vol.53, No.5, pp.649-654, 1989
Printed in Great Britain

0021-8928/89 $10.00+0.00

ON LIMIT SURFACE LOADS IN THE THEORY OF PLASTICITY*

O.O. BARABANOV

Within the framework of quasistatic plasticity theory, the specific features of surface tangential loading is demonstrated by simple examples: the possibility of a singular surface discontinuity, and the absence of convergence of limit load coefficients for an arbitrary unlimited diminution of the period of the plastic composite. The second singularity forces an acknowledgement that the hypothesis /1/ and its subsequent verification are false in the case of tangential surface loads.

1. Antiplane motions. Singular surface breakdown. We confine ourselves to the examination of rigidly plastic bimaterials in the antiplane and plane cases. The inhomogeneity will be given by using the periodic function $\tau(y)$, defined in the periodicity cell $Y = (0, 1)^2$ as follows

$$\tau(y) = \begin{cases} \tau_1, & y \in Y_k \\ \tau_2, & y \in Y \setminus Y_k \end{cases}$$

$$Y_k = \{y: \ |2y_i - 1| < k, \ i = 1, 2\}, \ 0 < \tau_1 \leqslant \tau_2, \ 0 < k < 1$$

*Prikl.Matem.Mekhan., 53, 5, 824-829, 1989